

## REVIEWS

**Mécanique Aléatoire (Random Mechanics).** By P. KREE and C. SOIZE. Dunod, 1983, 638 pp. FFr 220.

As stated in the Introduction, the authors' purpose in writing this book is to develop a closer association between mathematicians and applied users, in the domain of mechanical engineering where stochasticity is introduced via random external forces or random noise. The outline, based on the principle 'theory → applications → theory', contains three sections:

(i) A theoretical section A presenting the mathematical tools, some of which will be used in the applications of section B. These tools are linear and tensorial algebra, integration theory, probability theory, stochastic processes, spectral analysis and trajectory statistics (chaps. 1–5).

(ii) An applied section B, where the response of solid structures is considered under various random excitations: atmospheric turbulence (chap. 6), earthquakes (chap. 7), swells (chap. 8), random loading (chap. 9).

(iii) A theoretical section C, presenting further analytical methods in vectorial spaces, generalized random processes, stochastic differential equations, squared Gaussian processes (chaps. 10–15).

I will not pretend to discuss the mathematical content of this book; only a specialist in probability theory is qualified for that. My comments deal essentially with the second section B, which is appealing since it concerns extremely important practical applications. I was somewhat disappointed by the chapter devoted to atmospheric turbulence, where too much emphasis is given to the Van der Hoven spectrum, and where the turbulence is assumed to be Gaussian. Also I do not understand why the authors seem to need to neglect gravity in order to introduce the concept of turbulence: a constant gravity could be included in the pressure gradient. The only exact results concerning the action of turbulence upon structures are given in the case of 'elongated' structures (of characteristic dimension much smaller than the integral scale of turbulence). The method developed in the general case is difficult to follow and appears somewhat empirical. In addition, it is not clear why it works so well in the particular case of the tower Maine-Montparnasse in Paris. One can also ask if such powerful mathematics (11 chapters out of 15) is really necessary to attack a problem in which drastic (but certainly necessary) physical assumptions are made in order to propose a solution.

Chapter 7 on earthquakes was disappointing for similar reasons. A Fokker-Planck equation for a probability density is solved numerically by a Galerkin expansion in Hermite polynomials: by the end of the chapter the problem has been overwhelmed by the mathematics without any practical conclusion.

More generally, it is difficult to see 'the method of random mechanics' emerge in any of the mechanical problems considered, as the authors claim. Instead I was immediately and definitely lost by a huge amount of 'propositions' and 'theorems'. One wonders whether they are really necessary in random processes that are assumed to be Gaussian. The last chapters on Markovian techniques and stochastic differential equations contain mathematical results which could possibly improve the stochastic models of turbulence of Herring and Kraichnan (in *Statistical models and turbulence*,

Springer, 1972), or the methods of resolution of linear stochastic differential equations as developed by Brissaud and Frisch (*J. Math. Phys.* **15** (1974), 524–534) for instance. But these approaches are ignored by the authors. As a matter of fact, the bibliography of the book seems to be quite poor, at least for the chapters dealing with subjects I have some knowledge about. I was surprised in particular not to find any reference to O. M. Phillips's book on the dynamics of the upper ocean in the chapter devoted to water waves.

A further difficulty for readers of the *Journal of Fluid Mechanics* lies in the fact that the book is written in a French which is not always grammatically correct, and with somewhat unorthodox punctuation and spelling (see e.g. p. 293, 1 + 5).

The author of the present review is not a 'mathematician' (in the French sense), and is not qualified to comment upon the detailed mathematical developments and results contained in this book. Nevertheless I am not sure that such a monograph will contribute to fill the huge gap existing between 'theoretical mechanics à la Française' and 'Anglo-Saxon applied mathematics'. This book puts too much emphasis on a mathematical formalism which is unnecessary for physical problems we have not yet understood.

'Mécanique Aléatoire' is an attractive and ambitious title, but the content of the book does not correspond to its ambition. The reader may feel frustrated not to find any consideration of questions such as the nonlinear dynamics of turbulence, turbulent diffusion, acoustics, propagation in random media, or chaos in dynamical systems. Nevertheless, the probability-theoretician will certainly find this book extremely stimulating, and the physicist or the engineer will certainly use it as a basis for further research on questions of great practical importance concerning the action of random excitations on mechanical structures.

MARCEL LESIEUR

**Vortex Flow in Nature and Technology.** By HANS J. LUGT. Wiley Interscience, 1983. 297 pp. £47.45.

From the first sentence we are urged by an enthusiast to share his enthusiasm for vortices (at a price). The outcome is at once fascinating and frustrating. Fascinating for the tremendous variety of topics introduced, ranging from the motion of protozoa through the fall of raindrops, motions of boomerangs and frisbees, growth of snow cornices, voices of wind instruments, numerous aspects of aerodynamics, mountain winds, tornadoes and hurricanes, atmospheric and oceanic circulation and the Great Red Spot on Jupiter, to rotating stars, black holes and galactic vortices and much more. Frustrating because much of the treatment is superficial, and too little attention is addressed to the subtleties and difficulties of the subject. Critical judgement seems often to be suspended, and it is perhaps fair to ask whether the identification of coiled snail shells as a kind of vortex motion or the inclusion of three columns on plate tectonics adequately compensates in a book like this for the extreme vagueness on vorticity generation (with only the slightest mention of Lighthill's introductory chapters to Rosenhead's *Laminar Boundary Layers*) and the apparent absence of any mention of the vital coupling between axial and azimuthal velocity fields that plays so powerful a role in the behaviour of strong diffusive vortices.

Before analysing the contents further we should in fairness identify the author's target audience. According to the cover this is 'the first unified account of the vortex concept' which 'describes concisely the basic characteristics of vortices and distils the most significant work on vorticity dynamics from the vast literature...'. It is an

‘invaluable working tool for fluid dynamicists.... Meteorologists and oceanographers will appreciate [it]..., astronomers, biologists and historians will highly value [it]...’. Further, the preface recommends chapters 4–6 as ‘an unorthodox [student] introduction, based on vorticity dynamics, to the theory of viscous fluids’. This amounts to so exaggerated a prescription that our concern can only be in which parts, rather than whether, it is achieved. My view, in short, is that the book falls far short of a definitive technical analysis of vortices because of the author’s failure to identify and adequately explain a number of the subtler but essential properties of vortices and vorticity. I do not consider it suitable as a student introduction to viscous flow, because momentum and not vorticity is the primary variable in fluid mechanics, and in my experience arguments based wholly on vorticity obscure the fundamental dynamical relationship between force and rate of change in momentum and are generally puzzling until students have already some grasp of the subject; moreover, starting students have little basis for incorporating the great breadth of phenomena here presented qualitatively. It is not a coffee-table book, as the photographs prove to be fewer and smaller than a first impression suggests. Rather it is a sort of scientific ‘tearoom’ book, a very effective source book which one can open at random with a high probability of finding one or more interesting snippets, well referenced (with some weighting towards the German) and concisely presented.

The book is divided into two major parts covering vortices in a homogeneous irrotational environment, and those in rotating, sometimes stratified, environments, with a substantial appendix in which all mathematical treatment has been segregated, and a listing of some 770 references. A historical introduction leads into a chapter on kinetics where a vortex is defined as ‘the rotating motion of a multitude of material particles around a common centre’, a definition that includes concentrated vortices, rigid-body rotation and all regions of rotating fluid, no matter how extensive. This definition fails to distinguish between external and internal rotation, both of which constrain interior flow, but in quite different physical ways; equivalently, it does not distinguish between high- and low-Rossby-number flows. Lugt defines swirling flows in §5.6 as vortices with an axial velocity component, and in both chapters 3 and 5 discusses the example of solid-body rotation with a superposed uniform axial flow independent of radius. Neither here nor elsewhere does he so much as mention that axial flow is almost universal in strong core vortices (other than rings). This is, however, axial flow *contained within the vortex core*, and not external axial flow which merely advects the core without significant dynamical role, nor exterior convergence that may strengthen a weak vortex which will in turn develop its own internal axial velocity field. The axial flow of strong vortices is coupled through the pressure field of the vortex with its azimuthal velocity field, and axial gradients of pressure are generally related to diffusive development along the core. He continues in §5.6 to describe the effect of vortex termination at a fixed wall perpendicular to its axis, noting correctly that the ‘vorticity lines’ cannot end at the wall, but adding that they must spiral out in an Ekman boundary layer. This confusion is a consequence of the loose definition given earlier for a vortex: the interaction of a concentrated vortex with a boundary is an extremely important, but as yet only partially resolved problem, which Lugt neglects; in contrast the Ekman layer is a low-Rossby-number boundary flow in a system dominated by external rotation. Further, one does not expect to be told in a book on mechanics that a ball constrained by a string to move in a circular path is in equilibrium under the action of balanced centrifugal and centripetal forces (§3.2).

In §4.1 the author has two attempts to explain the generation of vorticity at

boundaries. (i) [In a fluid at rest] 'a body is now abruptly set in motion. The fluid is disturbed, and this disturbance spreads throughout the fluid. The pressure impulse travels with the speed of sound. Simultaneously, the fluid is sheared at the solid surface by the abrupt movement of the body. In this way, vorticity is generated.' Are you with me? (ii) 'Fluid particles do not slip along the wall but roll, owing to their adherence, like balls and cylinders in a bearing.' In fact a dye streak released from a hypodermic needle near a wall in a laminar boundary layer trails downstream as a line diverging slowly from the wall (and in Couette or Poiseuille flow remaining precisely parallel to it). Macroscopic particles of fluid are *not* rotating, and indeed the ballbearing analogy is an unhappy one because rate of strain plays a vital role in fluids, but (with luck) none at all in ballbearing races. The sections on aerodynamics and especially separation are among the more interesting of those on basic flows, but the introduction in §6.5 of a 'Rossby number' for flow of an incident stream past a rotating cylinder as the ratio of stream and cylinder-surface velocities shows a startling extension of normal usage.

The second part of the book deals with vortices in rotating and stratified systems and is rich in cases of interest. The presentation is descriptive and includes discussion of atmospheric and ocean circulation, of tornadoes, waterspouts, Langmuir vortices, tidal vortices and hurricanes, with a final chapter on vortices in planetary atmospheres and in astronomy. The identification of the earth as a solid-body vortex is, of course, permissible, but in a book on *vortex flow* does little to enhance understanding; it also tends to obscure the important difference that rotation in solids is specified by angular momentum, but in fluids by vorticity, since rotation in fluids is rarely related to specific axes. There is a difficulty, too, with the identification of Coriolis force (§7.2) with the conservation of angular momentum of a particle moving on the surface of the earth: how then does a particle moving east or west along a circle of latitude experience Coriolis force? Again, centrifugal force exists in rotating but not inertial frames, and the description of gradient flow in which air is continuously accelerated in a circular path of motion as a state in which 'the pressure, Coriolis, and centrifugal forces are in equilibrium' can only be regarded as careless. Enough! The references are a good, if rather personal selection, and have certainly added to my own lists; but to see the mathematics, which can do so much to aid understanding, relegated to a supplement is disappointing even if necessary for a proportion of readers (although an improvement on the German edition).

Some of my comments are sharp, but the author can be judged only on his chosen ground. I have tried to justify my view that this is no introduction for students (who are already muddled enough by vorticity without this help), nor yet a definitive text. However, taken as a romp through the author's private collection, I have both enjoyed the book and picked up several leads that had previously escaped me. But before I return to work, let me refill my cup with hot tea, stir it nonchalantly into rotation with sweeping strokes and pour a thin jet of cold milk into the centre of the cup: a modest vortex forms, shown by a surface dimple after pouring has stopped; if instead I pour boiling water, no dimple survives. Lugt has many more items to cull from the literature and I look forward to a further edition, with a much tighter and more careful treatment of fundamentals, clearer lines of development, and an even better collection of snippets, perhaps used a little more carefully to reinforce the lines of the argument.

BRUCE MORTON